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## Zero sound in liquid $^3\text{He}$

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**Abstract.** The velocity and attenuation of sound and zero sound have been measured at a frequency of 120 MHz. The velocity increases by  $5.6 \pm 0.3 \text{ m s}^{-1}$  over a temperature range centred on  $34 \pm 3 \text{ mK}$ , while the attenuation rises to a maximum value of  $700 \text{ cm}^{-1}$ . These results are comparable with those of Abel *et al.* at 15 and 45 MHz, and in agreement with the Landau theory with the parameter  $F_2$  approximately equal to zero. A comparison of our values for the velocity with our previous values for the impedance at 122 MHz shows that the fractional change in the impedance at the transition is larger than the fractional change in the velocity. This result appears inconsistent with the Bekarevich–Khalatnikov treatment of the impedance.

### 1. Introduction

The general features of zero sound predicted by Landau have been confirmed by measurements of the acoustic impedance of liquid  $^3\text{He}$ , and of the velocity and absorption of sound. There remains, however, one unexplained feature of the results. The velocity of zero sound measured at 15 and 45 MHz exceeds that of ordinary sound by about 3 per cent (Abel *et al.* 1966), this value being independent of the frequency as predicted by the Landau theory. On the other hand the observed increase in the value of the acoustic impedance amounts to  $10 \pm 3$  per cent at 1000 MHz (Keen *et al.* 1965) and  $7 \pm 1.7$  per cent at 122 MHz (Kirby and Wilks 1968). Although we would not expect the change in impedance to correspond exactly with the change in velocity, the theory of Bekarevich and Khalatnikov (1960) appears to imply that the change in the impedance should be *smaller* than that in the velocity (Brooker 1967). As no measurements of velocity and impedance have been made at the same frequency, we have measured the velocity and attenuation at 120 MHz in order to make a direct comparison with our previous measurements of the impedance.

### 2. Experimental procedure

The velocity and absorption of 120 MHz sound waves were measured simultaneously by observing the passage of  $1.5 \mu\text{s}$  pulses across a thin layer of liquid  $^3\text{He}$  between two piezoelectric quartz crystals. The sonic arrangements are shown diagrammatically in figure 1. The principal components are a long X cut crystal AB (10 mm in diameter and 30 mm long), and a thin X cut 40 MHz crystal C, mounted by an indium bond on an optically flat X cut quartz crystal D about 3 mm thick. The adjacent ends of the long and thin crystals were coated with layers of evaporated aluminium about  $0.1 \mu\text{m}$  thick to act as electrodes and as electrostatic shields. These two faces were separated by three spacers, made of aluminium foil approximately  $50 \mu\text{m}$  thick, and each of area about  $1 \text{ mm}^2$ . The spacers positioned the ends of the crystals parallel to each other, and also provided an electrical earth contact to both the plated surfaces. The long crystal was excited by a live electrode  $E_B$  in the form of

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a copper ring placed about 1 mm from the end of the crystal, while the indium bond was used as a live electrode  $E_C$  to receive signals from the thin crystal. The capacitances between each of these electrodes and earth formed parts of tuned circuits which matched the impedances of the crystals to those of 50  $\Omega$  coaxial feeder lines.

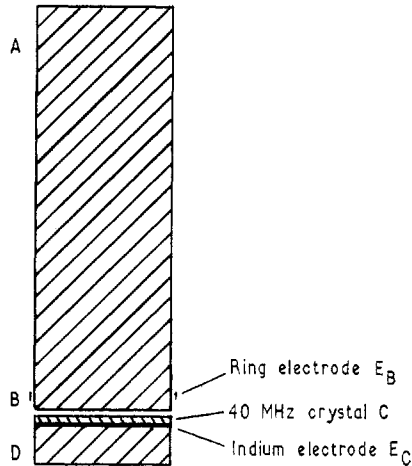


Figure 1. Schematic diagram of the acoustic arrangements.

To measure the velocity and absorption, a  $1.5 \mu\text{s}$  electrical pulse at a frequency of 120 MHz is applied to the ring electrode at the end B of the long crystal, thus creating a pulse of ultrasonic waves. A small part of the pulse passes through the liquid helium into the crystal C where it excites a signal in the second electrode. The majority of the sonic pulse in the crystal AB is reflected from the end B to the other (parallel) end of the crystal, and then travels up and down the crystal until it is completely absorbed. Each time it reaches the end B a small fraction is transmitted through the helium, and electrical signals are induced in the electrodes  $E_B$  and  $E_C$ . A comparison of the phase and amplitude of these induced voltages  $V_B$  and  $V_C$  then leads to values for the velocity and absorption.

To make the comparison, the two signals were passed through a two-channel superhet receiver, and converted to 30 MHz for ease of handling, using a common local oscillator. To obtain phase-sensitive detection two 30 MHz square-wave signals, differing in phase by  $\pi/2$ , were derived from the signal  $V_B$ . Each square wave was then used to chop both  $V_B$  itself and  $V_C$ , thus giving two pairs of voltages  $V_B'$ ,  $V_B''$  and  $V_C'$ ,  $V_C''$  which specify the amplitude and phase of the signals  $V_B$  and  $V_C$ .

In order to improve the signal-noise ratio, a synchronized and variable gating system selected about the middle third of the electric pulses, so as to avoid the lower amplitudes at the beginning and end of the  $1.5 \mu\text{s}$  pulses. This gating system was also used to verify that no errors in phase were resulting from multiple echoes in the helium. As the absorption becomes very high at the lower temperatures, the thickness of the layer of helium must be kept small and was therefore only about  $50 \mu\text{m}$ . Hence a second reflected pulse arrives at  $E_C$  only  $0.5 \mu\text{s}$  behind the initial pulse. In fact the amplitude of this next pulse was less by a factor of about ten because the faces of the long and short crystals were not exactly parallel. Moreover, only the initial (low-amplitude) portion of the second pulse overlapped the selected portion of the first

pulse, hence the effect of this second pulse should be small. This was confirmed by varying the length of the signal selected from the first pulse, and thus the amount of admixed signal from the second pulse. No change was detectable in the recorded phases and amplitudes.

As described above, the signal in the long quartz crystal travelled up and down many times before being absorbed, and the path in this crystal is so long that all the other sonic pulses died away during one round trip. Hence, each time the sonic pulse in the long crystal was reflected from the end B, it gave a pair of values of  $V_B$  and  $V_C$ . To use these to the best advantage the receiver was gated to detect the signals from the 5th to 25th pulses. The first four pulses were ignored to ensure that the receiver had fully recovered from the effects of the transmitter pulse, while the later signals were ignored to maintain a good signal-noise ratio. To record the measured voltages, the signals  $V_B'$ ,  $V_B''$ ,  $V_C'$  and  $V_C''$  were switched by FETs to four condensers where the integrated voltage gave a mean value for each signal. The main transmitter was pulsed at a repetition rate of about 0.2 Hz, and the steady voltages across the condensers plotted on a chart recorder.

To obtain values of the velocity from the phase measurements it was necessary to determine the exact thickness of the layer of helium by calibrating the apparatus with a liquid of known velocity. The relationship between the separation of the crystals  $d$ , the angular frequency of the sound  $\omega$ , the velocity of sound  $v$  and the observed phase difference  $\phi$  is

$$\phi = \frac{\omega d}{v} + \beta \quad (1)$$

where  $\beta$  is a constant but unknown phase shift in the quartz crystals. The system was calibrated by allowing the temperature to drift slowly, and the phase difference recorded continuously as the velocity changed. Using published values of the variation of velocity with temperature (Atkins and Flicker 1959, Laquer *et al.* 1959), graphs of  $\phi$  against  $1/v$  were plotted as shown in figure 2, which gives results obtained with  $^3\text{He}$  between 1.05 and 2.20 K. The good straight line verifies the relation (1) and gives the value of the spacing for the particular run in question to within 1%.

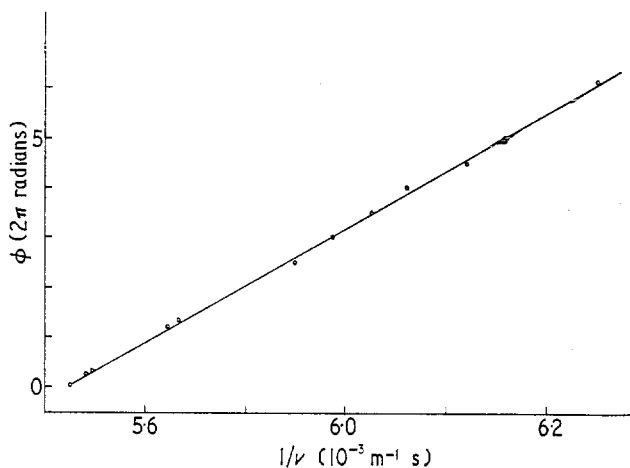


Figure 2. A calibration of the thickness of the layer of liquid  $^3\text{He}$  between 1.05 and 2.20 K.

Measurements of the spacing were also made with liquid  $^4\text{He}$  in the range 1.05 to 4.2 K and good straight plots were again obtained. However, figure 3 shows the results of another run with liquid  $^3\text{He}$  extending from 1.05 to 3.3 K. We see that the points above 2.20 K do not form a straight line with those below. We believe this to

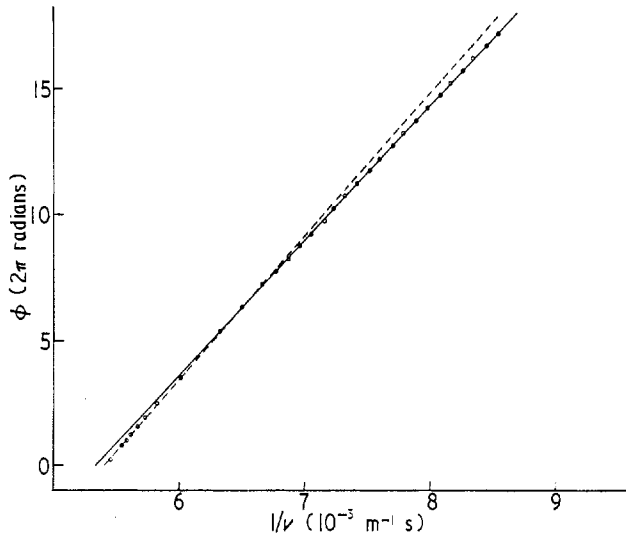


Figure 3. A calibration of the thickness of the layer of liquid  $^3\text{He}$  between 1.05 and 3.3 K.

indicate that the published values for the velocity are about  $2 \text{ m s}^{-1}$  low at 3.3 K. As these published results were extrapolated from measurements at somewhat higher pressures, this discrepancy is perhaps not surprising.

The liquid  $^3\text{He}$  and quartz crystals were cooled by thermal contact with 200 g of cerium magnesium nitrate, adiabatically demagnetized from a field of about 9 kG, commencing at a temperature of about 0.4 K. There was also a guard ring cooled by thermal contact with a demagnetized pill of chrome alum. The temperature of the liquid  $^3\text{He}$  was measured using a cerium magnesium nitrate frequency thermometer (Betts *et al.* 1964, Kirby and Wilks 1968), the salt being in direct contact with the experimental  $^3\text{He}$ . The salt occupied a cylindrical volume 4 mm in diameter and 12 mm long, and the thermometer was calibrated in the temperature range from 0.5 to 2.0 K.

### 3. Results

Two runs were performed in which the liquid  $^3\text{He}$  was cooled to below 25 mK. In both these runs the experimental chamber took approximately 4 hours to warm up to 0.1 K. There could, however, be a systematic error in the temperature between about 40 mK and 0.1 K as the warm-up rate in this range was comparable with the thermal equilibrium time between the thermometer and the experimental  $^3\text{He}$ . We estimate that, at the most, this would introduce an uncertainty of 3 mK.

Figure 4 shows how the velocity changed as a function of temperature below 0.3 K. The two sets of results are in good agreement. The method enables the phase shift to be measured to an accuracy of about 0.05 radians, which corresponds to an

error in the velocity of  $0.06 \text{ m s}^{-1}$ . The high absorption at the transition temperature causes a phase shift of the sound transmitted across the quartz- $^3\text{He}$  interface, but this has a maximum value of  $0.01$  radians and can be ignored.

At the lowest temperature, the velocity had not quite levelled off to a constant value. If we assume that the curve in figure 4 is symmetrical, the velocity will reach

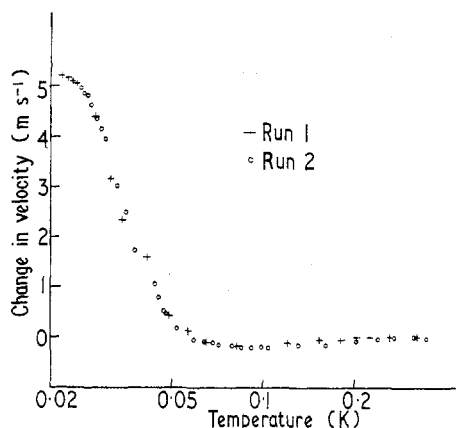


Figure 4. The change in the velocity of 120 MHz sound waves in liquid  $^3\text{He}$  below  $0.3 \text{ K}$ .

a constant value about  $0.2 \text{ m s}^{-1}$  greater than at our lowest temperature point. The total change below  $0.1 \text{ K}$  is then  $5.55 \text{ m s}^{-1}$ , with an uncertainty of  $0.3 \text{ m s}^{-1}$  arising principally from the uncertainty in the thickness of the specimen. Defining the transition temperature arbitrarily as the temperature corresponding to a velocity half way between the ordinary and zero sound limits, the average value for the two runs is then  $34 \pm 3 \text{ mK}$ .

The small rise in the velocity between  $0.1 \text{ K}$  and  $0.3 \text{ K}$  is believed to be due mainly to contamination of the sample by  $^4\text{He}$ . At very low temperatures liquid  $^3\text{He}$  and  $^4\text{He}$  separate into a pure  $^3\text{He}$  phase and a  $^4\text{He}$ -rich phase, so any  $^4\text{He}$  will sink to the

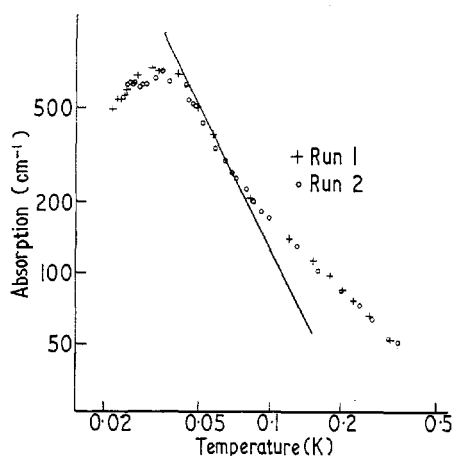


Figure 5. The coefficient of absorption of 120 MHz sound waves in liquid  $^3\text{He}$ .

bottom of the chamber where it will have no effect on the experiment. As the temperature rises the  $^4\text{He}$  will re-dissolve in the  $^3\text{He}$  and the mixture will show a higher velocity than pure  $^4\text{He}$ . We note in confirmation that in a separate run using a purer sample of  $^3\text{He}$  the rise in velocity between 0.1 and 0.3 K was only  $0.1 \text{ m s}^{-1}$ . It is estimated that a contamination of 0.4% would cause the observed effect.

Our results for the absorption are shown in figure 5. As our method measures changes in absorption rather than absolute values, we have chosen the absolute value between 0.5 and 1.0 K to agree with the absorption we would expect from the viscosity measurements of Betts *et al.* (1963). We observe a peak in the absorption curve at  $33 \pm 3 \text{ mK}$  with a value of  $700 \pm 70 \text{ cm}^{-1}$ . The theory predicts that at sufficiently low temperatures, on the high temperature side of the peak, the absorption should vary as  $\omega^2 T^{-2}$ . In fact our values never show so strong a dependence on temperature, as can be seen by comparison with the straight line in the figure corresponding to the relation  $\alpha = (2.30 \times 10^{-18})\omega^2 T^2$ . However, this discrepancy is not surprising, as the measurements of Abel *et al.* at lower frequencies show that the  $T^{-2}$  region is not reached until below 50 mK.

#### 4. Discussion

The change in velocity at 122 MHz ( $5.55 \pm 0.3 \text{ m s}^{-1}$ ) lies almost within the range reported by Abel *et al.* at 15 and 45 MHz ( $6.5 \pm 0.6 \text{ m s}^{-1}$ ). As would be expected from the theory, the transition to zero sound occurs at almost the same temperature as the change in the acoustic impedance (33 mK). We have previously shown this latter figure to be consistent with the transition temperatures reported by Abel *et al.* at 15 and 45 MHz.

If we assume that the viscosity varies as rapidly as  $T^{-2}$  only below about 50 mK, then we do not expect to observe an absorption with this temperature dependence over any appreciable range of temperature. However, the theory predicts that the peak absorption should be proportional to the frequency. Comparing the two available values, at 122 and 15 MHz, the ratio of the absorptions is 7.8, in good agreement with the ratio of the frequencies.

Our results for the velocity and absorption are essentially in accord with those of Abel *et al.*, and with the Landau theory taking the parameter  $F_2$  approximately equal to zero. As we have already noted, the fractional change in the impedance at the transition to zero sound is greater than the fractional change in the velocity. Our present results suggest that the source of this anomaly lies in the behaviour of the impedance.

It is important to rule out the possibility that the anomalous rise in the impedance is due to the generation of transverse waves of zero sound. For example, suppose that, in the ordinary sound region, a small fraction of the sonic energy passes into the liquid as transverse modes generated by the sides of the crystal. From the definition of the impedance (see for example Brooker 1967) and the equation of motion of transverse waves (Landau and Lifshitz 1959), it follows that the effective acoustic impedance  $Z_t$  for these modes is given by  $Z_t/\rho = (\omega\eta/2\rho)^{1/2}$ , where  $\omega$  is the angular frequency and  $\eta$  the viscosity. At a temperature just above the onset of the transition at 120 MHz this gives a value of  $Z_t/\rho$  of about  $20 \text{ m s}^{-1}$ . The Landau theory predicts that the velocity of any transverse zero sound will be of the order of the Fermi velocity, say about  $60 \text{ m s}^{-1}$  (see for example Corruccini *et al.* 1969). If we now assume, by analogy with the behaviour of longitudinal waves, that the associated acoustic impedance in the zero sound region is approximately equal to the product of the density

and the transverse wave velocity, then the value of  $Z_t/\rho$  in the zero-sound region should increase to about  $60 \text{ m s}^{-1}$ . On passing into the zero sound region, the energy dissipated in transverse modes will increase in proportion to the increase in the impedance  $Z_t$ , that is by a factor of about three. The total change in the impedance observed in an experiment will then exceed that to be expected if only longitudinal modes are excited in the liquid.

The above figures show that the impedance measured in our experiment may be considerably enhanced if a small fraction of the energy is dissipated in transverse modes. The anomalously large value of  $Z/\rho$  at 122 MHz could be accounted for if only 2% of the energy dissipation in the liquid, at a temperature just above the transition, takes place via transverse modes. However, we believe that any such effect is insignificant. If we assume, for example, that 2% of the energy is dissipated in this way, we would then expect the value of  $Z/\rho$  at 0.05 K to be the sum of two components:  $183 \text{ m s}^{-1}$  associated with the longitudinal modes and about  $3.5 \text{ m s}^{-1}$  associated with the transverse modes. In the normal sound region the transverse contribution is proportional to the square root of the viscosity; hence, taking measured values of the viscosity we find that a contribution of  $3.5 \text{ m s}^{-1}$  should decrease to  $1.6 \text{ m s}^{-1}$  at 0.15 K. Figure 6 of our previous paper (Kirby and Wilks 1968) shows values of the change in  $Z/\rho$  measured from a reference level corresponding to the value at about 0.15 K. There is no sign of an increase of about  $1.6 \text{ m s}^{-1}$  between 0.15 and 0.05 K which we would expect if transverse modes were present.

In a previous experiment at 1000 MHz (Keen *et al.* 1965) we observed values of  $Z/\rho$  which were about  $13 \text{ m s}^{-1}$  above the velocity of zero sound. To account for this value by the presence of transverse modes would require the transverse dissipation in the normal region, at a temperature just above the transition, to contribute at least  $4 \text{ m s}^{-1}$  to the value of  $Z/\rho$ . We would then expect the measured impedance to have a value of at least  $187 \text{ m s}^{-1}$ . However, figure 7 of Keen *et al.* shows that the measured value is clearly lower, and close to the expected value of  $183 \text{ m s}^{-1}$ .

There are also other grounds for supposing that the effects of transverse modes will be insignificant. Because of the relatively low acoustic impedance associated with transverse modes, even small dissipations imply quite large oscillatory displacements of the sides of the crystal. Taking into account the different areas of the sides and ends of the crystals, and the length of the sonic pulse, a 2% transverse dissipation at 0.05 K implies that the oscillatory displacements of the sides of the crystal must be about 10% that of the ends. This seems unreasonably large as the usual (Pochhammer) solutions for the propagation of waves in rods have nodes on the side surfaces.

We conclude that the anomalously high value of the increase in  $Z/\rho$  at the transition to zero sound remains unexplained, and there appears to be some factor involved which is not yet appreciated. This position recalls the discrepancies between the measured values of the thermal boundary resistances between liquid helium and solid dielectric surfaces and values calculated by considering the propagation of the thermal waves across the boundary (see for example Wilks 1967). This latter problem is complicated by the well known difficulty of preparing solid surfaces with a known specification. In fact, one of the most striking features of our measurements of the impedance at both 122 and 1000 MHz is that the values obtained in the region of ordinary sound are exactly what we expect for a geometrically ideal crystal with no complications of surface roughness! We are continuing our experiments by making measurements at 10 GHz to obtain more information on the value of the impedance, as a function of frequency.



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